

Towards an hybrid compactification with a scalar-tensor global cosmic string

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Abstract

We derive a solution of the gravitational equations which leads to a braneworld scenario in six dimensions using a global cosmic string solution in a low energy effective string theory framework. The final spacetime is composed by one warped brane with $\mathbb{R}^{(3,1)} \times S^1$ topology and a power law warp factor, and one noncompact extra dimension transverse to the brane. By looking at the current experimental bounds, we find a range of parameters in which, if the on-brane dimension has an acceptable size, it does not solve the hierarchy problem. In another example this problem is smoothed by the Brans-Dicke parameter.

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I. INTRODUCTION

In the recent years the amount of works dealing with the possibility of large extra dimensions in the universe was crescent [1, 2]. The general characteristic of these type of models is to treat our universe as a four dimensional brane (a 3-brane) in a higher dimensional spacetime [3]. In the Randall-Sundrum model [4], a $5 - D$ realization of the Horava-Witten solution [5], the hierarchy problem can be solved by introducing an appropriated exponential warp factor in the metric. The general structure of the Randall-Sundrum model is given by two mirrors domain walls (the branes) embedded into an orbifold in the extra noncompact directions. Following the approach of the use of topological defects as the generator of bulk-brane structure, an alternative scenario was proposed in [6] in which a $6 - D$ spacetime comes from a global cosmic string in the Einstein gravity. These type of models are characterized by the global long range signature of the gravitational field, which restricts the compactification possibility of extra dimensions.

The program of compactification in General Relativity was extended to more dimensions using global defects [7]. In the reference [8], for instance, a higher codimension case is analyzed where the hierarchy is introduced by hand, with a power law warp factor. Besides, the scenario presents an important characteristic revealed by the shape of the spacetime: far from the brane, the geometry is almost unaffected by the presence of the brane. On the other hand the backreaction effects are important near the brane. In this case, a naked singularity is expected in the zero width limit.

Frequently, in the braneworld framework it is necessary to introduce scalar fields in the bulk in order to localize the branes [9, 10]. Apart of this, advances in string theory point into a presence of a dilatonic scalar field in the low energy spectrum of recovered gravity [11]. In this context it seems natural to work in theories such as the Brans-Dicke one [12], which is the easiest scalar-tensor gravity theory, having essentially a scalar field intermediating the gravitational force together with the usual purely tensor field $g_{\mu\nu}$. In such framework, a braneworld model using a local cosmic string was proposed in [13], however due to the complicated functional form of the field equations, the main argumentation was based on dynamical systems tools.

The main purpose of the present paper is to extend ideias of the ref. [13] and to present a solution of the Einstein-Brans-Dicke (EBD) equation which points into an exotic (and hybrid) compactification. Here, we use a global cosmic string model coupled to gravity. Since the establishment of a braneworld model is, at least initially, mainly a classical issue, we focus our arguments in the gravitational aspects of the Brans-Dicke theory. We start with a $(p + 3)$ -dimensional set up, looking for a solution of the EBD system with the energy-momentum of a global cosmic string *outside* of the string core. We do not consider the mechanism of string formation. Instead, we just establish the spacetime outside the string core. As usual for this type of defect, the bulk-brane structure obtained here has six dimensions. This type of six-dimensional scenario was previously studied in General Relativity with an explicit local vortex source [14]. Besides, a blown-up brane was analyzed in [15, 16], where a \mathbb{Z}_2 symmetry connects the solutions inside and outside the brane. Returning to the present work, after solving the EBD equation we restrict the range of integrations constants in order to study some specific examples.

The final scenario appears to be strongly dependent on the relationship between such constants and our choice is made in order to analyze a concrete example. If, by one side, a concrete case provides a more insightful situation from the physical point of view, from another side we pay the price of obtaining conclusions which are restricted to the case in question. Nevertheless, we should keep the study of specific examples, since in this way we can perform a more critical

analysis. The line element in such examples shows, then, an hybrid compactification with one 4-brane with topology given by $\mathbb{R}^{(3,1)} \times S^1$ and a power law warp factor. The spacetime far from the brane is not well-behaved, as well as in the limit of the transverse dimension going to the brane. We remark, however, that the solution is obtained for $r \geq r_b$, where r is codimension and $r_b(> 0)$ is the string radio (the brane width). This last characteristic is, in some sense, similar to the case found in the reference [8] for higher codimension braneworld in General Relativity.

The final result also shows an important relation between the radius of the extra on brane dimension and the warp factor: in one example, as the warp factor decreases with the transverse dimension, the extra dimension size grows, and vice-versa. Such a behavior is not new in Brans-Dicke braneworld models [9]. In another example, this opposite behavior remains but it is slightly modified. In reference [13], it was clear that the Brans-Dicke scalar field opens new possibilities of adjustment in the solution of the hierarchy problem via the warp factor. Nevertheless, by looking at the current lower bound of the Brans-Dicke parameter [17], we find a range of parameters in which the extra on-brane dimension has an acceptable size, but it does not solve the hierarchy problem in this (pre)model. Such a characteristic is also present in other models [15]. In the least example analyzed here it appears a more interesting warp factor, in which the hierarchy problem is smoothed by the Brans-Dicke parameter.

The paper is organized as follows. First, we generalize the solution for a global string in four dimensions obtained by Boisseau and Linet in [18] for six dimensions, starting from $(p+3)$ -dimensions and after setting $p=3$ in order to recognize a bulk-brane set up. Then we work in the sense of constrain the range of parameters in order to study some concrete examples in this pre-model. Finally, in the last section we conclude summarizing our main results and discussing the (still) open issues to be considered in the future, as the generalization of the solution to $r \leq r_b$ and the moduli stabilization, both highly necessary in a realistic model.

II. TOWARDS THE MODEL

We shall first analyze a global cosmic string model in $(p+3)$ -dimensions within the Brans-Dicke gravity (hereafter called simply as BD). Then, we fix the dimension in order to recognize the bulk-brane structure. It is more convenient to work with the non-physical metric, i.e., in the Einstein's frame, where the tensorial field $g_{\mu\nu}$ decouples from the scalar field (the "dilaton") ϕ , related to the physical one by $\tilde{g}_{\mu\nu} = e^{2\alpha\phi} g_{\mu\nu}$. By definition, the physical BD scalar field is $\tilde{\phi} = \frac{1}{\mathcal{G}} e^{-2\alpha\phi}$, where α is a dimensionless parameter related to the BD one, w , by $\alpha^2 = \frac{1}{2w+3}$ and \mathcal{G} is correlated to the Planck mass in $(p+3)$ dimensions.

The ansatz for the non-physical metric will respect the symmetry produced by a straight $U(1)$ global cosmic string, i. e., a cylindrical symmetry

$$ds^2 = -g(r)dt^2 + dr^2 + g_1(r)d\theta^2 + g(r)dz_i^2, \quad (1)$$

where $i=1\dots p$. Besides, outside the string core radius, the stress tensor is given by [18]

$$T_r^r = T_z^z = T_t^t = -T_\theta^\theta = -\sigma(r), \quad (2)$$

where σ is a strictly positive function to be determined. Note that with such settings we are considering the brane at the origin and the solution will be valid for $r \geq r_b$, where r_b is the string radius core (the brane width).

In terms of $g_{\mu\nu}$ the BD equation is

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}g^{\gamma\delta}\partial_\gamma\phi\partial_\delta\phi + 8\pi\mathcal{G}T_{\mu\nu}, \quad (3)$$

or, in a more convenient way,

$$R_\nu^\mu = 2\partial^\mu \phi \partial_\nu \phi + 8\pi \mathcal{G} \left(T_\nu^\mu - \frac{\delta_\nu^\mu T}{p+1} \right) \quad (4)$$

and the scalar BD field satisfies the wave equation

$$\square \phi = -4\pi \mathcal{G} \alpha T. \quad (5)$$

The stress tensor is no longer conserved in this frame. In fact,

$$\nabla_\mu T_\nu^\mu = \alpha T \partial_\nu \phi, \quad (6)$$

which is equivalent to

$$\frac{\sigma'}{\sigma} + \frac{g_1'}{g_1} + \frac{(p-1)}{2} \frac{g'}{g} = 2\alpha \phi'. \quad (7)$$

Its solution is given by

$$\sigma = \frac{\sigma_0}{8\pi \mathcal{G}} \frac{e^{2\alpha \phi}}{g_1 g^{\frac{(p-1)}{2}}}, \quad (8)$$

where σ_0 is an arbitrary positive constant. From equation (5), and using (8), one has

$$\phi'' + \frac{1}{2} \left[\frac{g'(p+1)}{g} + \frac{g_1'}{g_1} \right] \phi' = \frac{\alpha \sigma_0 e^{2\alpha \phi}}{g_1 g^{\frac{(p-1)}{2}}}. \quad (9)$$

The set of equations (4), with the metric (1), gives

$$\frac{-g''}{2g} + \frac{(1-p)g'^2}{4g^2} - \frac{g'g_1'}{4gg_1} = 8\pi \mathcal{G} \sigma \left(\frac{1-p}{1+p} \right), \quad (10)$$

$$\frac{-g_1''}{2g_1} + \frac{g_1'^2}{4g_1} - \frac{(p+1)g_1'g'}{4g_1g} = 8\pi \mathcal{G} \sigma \left(\frac{3+p}{1+p} \right), \quad (11)$$

$$-\frac{(p+1)}{2} \frac{g''}{g} + \frac{(p+1)}{4} \frac{g'^2}{g^2} - \frac{g_1''}{2g_1} + \frac{g_1'^2}{4g_1^2} = 8\pi \mathcal{G} \sigma \left(\frac{1-p}{1+p} \right) + 2\phi'^2. \quad (12)$$

Now, defining $\bar{u}^2 = g_1 g^{(p+1)}$, one can rewrite the equations (9)-(11), respectively, as

$$\frac{d}{dr} \left(\bar{u} \frac{d\phi}{dr} \right) = \alpha \sigma_0 \bar{u} \frac{e^{2\alpha \phi}}{g_1 g^{\frac{(p-1)}{2}}}, \quad (13)$$

$$\frac{d}{dr} \left(\frac{\bar{u}}{g} \frac{dg}{dr} \right) = -16\pi \bar{u} \sigma \left(\frac{1-p}{1+p} \right), \quad (14)$$

$$\frac{d}{dr} \left(\frac{\bar{u}}{g_1} \frac{dg_1}{dr} \right) = -16\pi \bar{u} \sigma \left(\frac{3+p}{1+p} \right). \quad (15)$$

Substituting the equations (10) and (11) into (12) we have

$$\frac{p(p+1)}{4} \frac{g'^2}{g^2} + \frac{(p+1)}{2} \frac{g_1'g'}{g_1g} = 8\pi \mathcal{G} \sigma (p-3) + 2\phi'^2. \quad (16)$$

It is useful to introduce a new radial coordinate, \bar{r} , just like in [18] by defining $\bar{u} \frac{d\bar{r}}{dr} = 1$. The new coordinate runs in the domain $-\infty < \bar{r} < +\infty$, and in terms of this new variable the metric becomes (suppressing the bar label)

$$ds^2 = -g(r)dt^2 + g^{(p+1)}g_1dr^2 + g_1(r)d\theta^2 + g(r)dz_i^2. \quad (17)$$

Note that in this new coordinate system the brane is located at $r \rightarrow -\infty$ and the solutions will be valid in the region $r \geq -r_b$. Rewriting the equations (13)-(15), we have

$$\frac{d^2\phi}{dr^2} = \alpha\sigma_0 g^{\frac{(p+3)}{2}} e^{2\alpha\phi}, \quad (18)$$

$$\frac{d}{dr} \left(\frac{1}{g} \frac{dg}{dr} \right) = \frac{-2(1-p)\sigma_0 g^{\frac{(p+3)}{2}} e^{2\alpha\phi}}{(1+p)}, \quad (19)$$

$$\frac{d}{dr} \left(\frac{1}{g_1} \frac{dg_1}{dr} \right) = \frac{-2(p+3)\sigma_0 g^{\frac{(p+3)}{2}} e^{2\alpha\phi}}{(1+p)}, \quad (20)$$

while the equation (16) remains the same, apart of a \bar{u}^2 term multiplying the first part of the right-hand side.

From the equations (18) and (20) we obtain

$$g_1 = g_1^0 e^{\kappa_1 r} e^{\frac{-2}{\alpha} \left(\frac{3+p}{1+p} \right) \phi}. \quad (21)$$

Equations (18) and (19) together show that

$$g = g^0 e^{\kappa r} e^{\frac{-2}{\alpha} \left(\frac{1-p}{1+p} \right) \phi}, \quad (22)$$

with κ_1 and κ being arbitrary constants while g_1^0 and g^0 are positive constants. We should emphasize that, if $p = 1$, the expressions for g and g_1 agree with the solutions found in [18]. Looking at the expression (17), it is clear that if $p = 3$ we arrive into a braneworld scenario. From now on, we shall fix $p = 3$, i.e., we are dealing with a six dimensional model. Substituting the expressions obtained for g and g_1 into equation (16) one has

$$\left(2 + \frac{3}{\alpha^2} \right) \phi'^2 - \frac{2\kappa_1}{\alpha} \phi' - \kappa \left(3\kappa + 2\kappa_1 \right) = 0, \quad (23)$$

which solution is given by $\phi = \phi_0 r + \phi_1$, where ϕ_1 is an arbitrary constant (hereafter $\phi_1 = 0$ without loose of generality) and

$$\phi_0 = \frac{\kappa_1}{\alpha \left(2 + \frac{3}{\alpha^2} \right)} \pm \frac{1}{2 \left(2 + \frac{3}{\alpha^2} \right)} \left[\frac{4\kappa_1^2}{\alpha^2} + 4\kappa (3\kappa + 2\kappa_1) \left(2 + \frac{3}{\alpha^2} \right) \right]^{1/2}. \quad (24)$$

With ϕ one can determine exactly the behavior of g_1 and g functions and then we can write down the expression for the metric. The physical line element for the spacetime is

$$\begin{aligned} d\tilde{s}^2 = & g^0 e^{\left[\kappa + \left(\frac{1+2\alpha^2}{\alpha} \right) \phi_0 \right] r} \eta_{\mu\nu} dx^\mu dx^\nu + g_1^0 e^{\left[\kappa_1 + \left(\frac{-3+2\alpha^2}{\alpha} \right) \phi_0 \right] r} d\theta^2 \\ & + (g^0)^4 g_1^0 e^{\left[4\kappa + \kappa_1 + \left(\frac{1+2\alpha^2}{\alpha} \right) \phi_0 \right] r} dr^2. \end{aligned} \quad (25)$$

Besides, the physical BD field is given by

$$\tilde{\phi} = \frac{1}{g} e^{-2\alpha\phi_0 r}. \quad (26)$$

It is well known that the solution for the global cosmic string leads to singularities in the spacetime, both in General Relativity [19] and in BD theory [18]. This effect is, in part, a characteristic of the dimension in question. From the equation (16) it is easy to see that, if the dimension is not six ($p \neq 3$) the first term of the right-hand side is not zero. This term leads to a differential equation other than (23). For example, in the Boisseau and Linet work ($p = 1$) the singularity for a finite value of r is present. This singularity comes from a contribution of such term. Fortunately, in the present case it is suppressed. However the equation (25) is not free of singularities. For example, the Ricci scalar is given by

$$R = \frac{[5\phi_0^2(4\alpha^2 - 1) - \alpha^2\kappa(3\kappa + 2\kappa_1) - 2\alpha\kappa_1\phi_0 + 8\phi_0^2]}{(g^0)^4 g_1^0 \alpha} e^{-\tilde{\sigma}r}, \quad (27)$$

which means that, if the coefficient $\tilde{\sigma} \equiv 4\kappa + \kappa_1 + \left(\frac{1+2\alpha^2}{\alpha}\right)\phi_0$ of r in the exponential factor is positive, the spacetime have a singularity when $r \rightarrow -\infty$. Of course, if the coefficient is negative the same thing happens to $r \rightarrow +\infty$. This cumbersome situation can be solved if we split the radial direction in two points, say $r = -r_0$ and $r = r_c$, placing a new brane at each end points [20].

We will, however, take another direction by imposing an additional constraint in the integration constants in order to have some specific cases of analysis.

III. SOME SPECIFIC EXAMPLES

The line element found in equation (25) is very general outside the string core. Potentially it can lead to a very interesting variation of the current braneworld models. In this section, we study some examples by imposing constraints in the integration constants. Our hope is that they would serve as good start points to more realistic models using global cosmic strings in scalar-tensorial gravity theories.

As one can imagine, the choice of such constraints should be done carefully. Let us analyze two examples where this choice can be problematic. The first restriction one can, eventually, try is to impose $\tilde{\sigma} = 0$. This case appears very interesting since it leads to a constant curvature scalar, i. e., a maximally symmetric spacetime. Apart of this, it is easy to note from (25) that such an imposition provides “naturally” an exponential warp factor. However, this constraint is not possible to be implemented. In fact, with the help of the equation (24) one can relate κ and κ_1 by

$$(20\alpha^2 + 45 - 12\alpha^4)\kappa^2 + (24\alpha^2 + 30 - 8\alpha^4)\kappa_1\kappa + (6\alpha^2 + 5)\kappa_1^2 = 0. \quad (28)$$

Now, we can restrict the values of the constants κ and κ_1 by looking at the experimental bound of the Brans-Dicke parameter. The current lower bound of the parameter w set it by $w \sim 10^4$ [17]. Hence, the most important terms are proportional to α^2 and there is not a solution for κ and $\kappa_1 \in \mathbb{R}$. The same problem appears, for instance, if the second term of the right-hand side of equation (24) vanishes.

Let us remark an important assumption here. The bound to the Brans-Dicke parameter is achieved from Solar System experiments. Here, we are assuming that it is unchanged in a braneworld scenario. In other words, we are assuming that the status of the Brans-Dicke theory is also valid in the bulk. It is a strong assumption. A complete scenario must take into account the solution inside the brane, where this assumption is largely valid, and then extend the result to the metric (25) by some junction conditions. In what follows, we still work with the hypothesis that

we can extrapolate the bound of w to the brane surface¹ $r = -r_b$ and to the bulk.

Since it is not possible to work with the hypothesis that $\tilde{\sigma} = 0$, let us take another one, this time with a more direct relation between κ and κ_1 . It can be done by

$$3\kappa + 2\kappa_1 = 0. \quad (29)$$

In this case, two solutions are possible. From equation (24) one finds

$$\phi_0^\pm = \frac{-3\kappa(1 \pm 1)}{2\alpha(2 + 3/\alpha^2)}. \quad (30)$$

Calling the exponential factors in equation (25) of

$$A \equiv \kappa + \left(\frac{1 + 2\alpha^2}{\alpha}\right)\phi_0, \quad (31)$$

$$B \equiv \kappa_1 + \left(\frac{-3 + 2\alpha^2}{\alpha}\right)\phi_0, \quad (32)$$

and

$$C \equiv 4\kappa + \kappa_1 + \left(\frac{1 + 2\alpha^2}{\alpha}\right)\phi_0, \quad (33)$$

we have, in the light of equations (29) and (30), the following cases

$$A^\pm = \kappa \left[1 - \frac{3(1 \pm 1)}{2} \left(\frac{1 + 2\alpha^2}{3 + 2\alpha^2} \right) \right], \quad (34)$$

$$B^\pm = \frac{-3\kappa}{2} \left[1 + (1 \pm 1) \left(\frac{-3 + 2\alpha^2}{3 + 2\alpha^2} \right) \right], \quad (35)$$

and

$$C^\pm = \frac{\kappa}{2} \left[5 - 3(1 \pm 1) \left(\frac{1 + 2\alpha^2}{3 + 2\alpha^2} \right) \right]. \quad (36)$$

The scalar curvature has also a split into “plus” and “minus” solutions given by

$$R = \frac{9\kappa^2(1 \pm 1)e^{-Cr}}{2(g^0)^4(g_1^0)(2\alpha^2 + 3)} \left[\frac{(1 \pm 1)}{2} [8 + 5(4\alpha^2 - 1)] - 1 \right]. \quad (37)$$

Now let us consider these two cases separately. The simplest one is given by ϕ_0^- . In that case we have $A = \kappa$, $B = -3\kappa/2$, $C = 5\kappa/2$, $R = 0$ and $\phi_0 = 0$ (the physical Brans-Dicke field is “unplugged”). Hence, the metric is simply given by

$$\tilde{ds}^2 = e^{\kappa r} g^0_{\mu\nu} dx^\mu dx^\nu + g_1^0 e^{\frac{-3\kappa r}{2}} d\theta^2 + (g^0)^4 g_1^0 e^{\frac{5\kappa r}{2}} dr^2. \quad (38)$$

In order to give the precise role of the warp factor we change again the coordinates. So, by the relation

$$d\rho^2 = \bar{C}_1 e^{Cr} dr^2, \quad (39)$$

¹ We currently understand a point $(x^\mu, \theta, r = -r_b)$, as belonging to the brane “surface”.

where $\bar{C}_1 = (g^0)^4(g_1^0)$, we have

$$e^{\xi r} = \left(\frac{C\rho}{2\bar{C}_1^{1/2}} \right)^{2\xi/C}, \quad (40)$$

where ξ is some constant. Note that the new coordinate system runs in the range $\rho_b < \rho < +\infty$ where ρ_b labels the brane surface. Now, the line element (38) reads (absorbing g^0 and g_1^0 factors)

$$\tilde{d}s^2 = \left(\frac{5\kappa\rho}{4} \right)^{4/5} \eta_{\mu\nu} dx^\mu dx^\nu + \left(\frac{5\kappa\rho}{4} \right)^{-6/5} d\theta^2 + d\rho^2. \quad (41)$$

The first thing to note from the above equation is the profile of the line element. It is not well defined in the limit of zero width $\rho_b \rightarrow 0$. Besides, the power law warp factor shows a problematic behavior as $\rho \rightarrow +\infty$. We emphasize that, apart of the fact that $R = 0$ (and $R_{ab}R^{ab} = 0$, $a, b = 0, \dots, 5$), we expected a naked singularity in the zero width limit since $R_{abcd}R^{abcd} \propto 1/\rho^4$ (and also $C_{abcd}C^{abcd} \propto 1/\rho^4$). To finalize the analysis of this case, let us study the metric (41) on the brane surface $\rho = \rho_b$.

Note that the warp factor of $d\theta^2$ in (41) is not zero when $\rho = \rho_b$, the brane surface. So, keeping the braneworld spirit in mind, we are forced to conclude that the system consists of an extra transverse dimension and a 4-brane with a compact curled extra dimension on-brane [9]. This means that the system presents an hybrid compactification². The spacetime described by (41) contains, then, a 4-brane with $\mathbb{R}^{(3,1)} \times S^1$ topology placed in $\rho = 0$ but, about this, we do not have any information of the internal structure. It is interesting the opposite behavior of the exponential terms in (41): for instance, as the warp factor decreases with ρ , the size of the curled dimension increases, and vice-versa. This behavior is also present in other models in the Brans-Dicke gravity framework [9].

The size of the curled on-brane extra dimension should not contradict the experimental bound [21], say M . So, from (41) it is implemented by

$$\left(\frac{5\kappa\rho_b}{4} \right)^{-6/5} < \frac{M}{2\pi}, \quad (42)$$

which means that

$$\rho_b > \frac{4}{3\kappa} \left(\frac{2\pi}{M} \right)^{5/6}. \quad (43)$$

The constraint (43) shows up the inviability of the solution of the hierarchy problem, since a huge warp factor amplifies it. In fact, the scale of the gravity interaction on the brane, say M_6 , is given by

$$M_6^4 = \frac{M_{Pl}^2}{2\pi} \left(\frac{4}{3\kappa} \right)^{6/5} \rho_b^{6/5}, \quad (44)$$

where M_{Pl} is the four dimensional Planck mass. Hence, taking into account the relation (43), one sees that $M_6 \gg M_{weak}$ and the hierarchy problem persists.

This type of behavior is also presented in other models [15]. Of course, it is not a desirable characteristic for braneworld models. However, we should emphasize that it is not a final word since

² On the other hand, if the warp factor of the θ dimension presents an infinity number of zeros, it will be possible to place infinity branes at each of these points [9] in the extra dimensional space. Note that, according to this interpretation, questions about the behavior of the θ dimension outside the branes are meaningless. In other words, the fact that the warp factor of $d\theta^2$ is able to take any value in the range of ρ is just important to fix the size of this on-brane dimension.

it depends explicitly of our previous choice of κ_1 and κ_2 . Besides, we do not have any information about the brane structure and it is possible that the shape of the warp factor changes in the regime $\rho < \rho_b$.

Let us study one another possibility, ϕ_0^+ . As we will see, this solution shows some interplay with the previous case in the sense that the main conclusions are basically the same, apart of a more interesting warp factor. In the case ϕ_0^+ we have in the old coordinate system $A = \kappa \left[1 - 3 \left(\frac{1+2\alpha^2}{3+2\alpha^2} \right) \right]$, $B = \frac{-3\kappa}{2} \left[1 + 2 \left(\frac{-3+2\alpha^2}{3+2\alpha^2} \right) \right]$, $C = \frac{\kappa}{2} \left[5 - 6 \left(\frac{1+2\alpha^2}{3+2\alpha^2} \right) \right]$, $R = \frac{9\kappa^2(20\alpha^2+2)}{(g^0)^4(g_1^0)(2\alpha^2+3)} e^{\frac{-\kappa(9-2\alpha^2)r}{2(3+2\alpha^2)}}$ and $\phi_0 = \frac{-3\kappa\alpha}{3+2\alpha^2}$. Now, with the help of equations (39) and (40) we arrive into the following line element (absorbing the g^0 , g_1^0 , κ factors and some coefficients proportional to the Brans-Dicke parameter)

$$d\tilde{s}^2 = \rho^{\left(\frac{-16\alpha^2}{9-2\alpha^2}\right)} \eta_{\mu\nu} dx^\mu dx^\nu + \rho^{\left(\frac{18(1-2\alpha^2)}{9-2\alpha^2}\right)} d\theta^2 + d\rho^2. \quad (45)$$

Note that the relation between the warp factor and the $d\theta$ coefficient as ρ changes is different from what we found in the previous case. These two terms still have opposite behavior, but now the exponent of the warp factor is negative, while the exponential factor of the $d\theta$ coefficient is positive³. The scalar curvature depends of ρ as $R \propto 1/\rho^2$. So, we again expect a naked singularity in the regime $\rho_b \rightarrow 0$.

By the same reason we discussed in the previous case, here the final scenario is composed by a $\mathbb{R}^{(3,1)} \times S^1$ brane and a transverse non-compact dimension. The solution on the brane shows a more interesting case in which concerns to the hierarchy problem. Restricting the size of the on-brane extra dimension we have

$$\rho_b < \left(\frac{M}{2\pi} \right)^{(9-2\alpha^2)/9(1-2\alpha^2)}. \quad (46)$$

However, this very small value for ρ_b can be, at least in part, compensated by the minuteness of the Brans-Dicke parameter and the warp factor appears not so huge.

To finalize this Section we stress that the physical Brans-Dicke field obeys the following power law $\tilde{\phi} \sim \rho^{24\alpha^2/(9-2\alpha^2)}$ which, in the light of equation (46), is very weak on the brane. We shall comment about the main results in the next Section.

IV. FINAL REMARKS AND OUTLOOK

To start with, we solved the EBD gravitational equations using a global cosmic string as a source in six dimensions. The topology of the brane is given by $\mathbb{R}^{(3,1)} \times S^1$, i. e., an hybrid compactification. Of course, the topology of the bulk is not a direct product of $\mathbb{R}^{(3,1)} \times S^1$ and some extra dimension geometry since the dimensions are mixed up [22]. An important issue would be to extend this topology to encode an orbifold in both extra directions. In other words, to replace the on brane S^1 by S^1/\mathbb{Z}_2 and try to incorporate this orbifold in the non-compact dimension too. In such case the matching conditions are well established and one can analyze some interesting properties of physical systems [23] such as black-holes on the brane.

Going forward, in order to extract some physical conclusions, we particularized a specific range for the integration constants in order to study some specific concrete examples. This analysis

³ Of course, here we are again assuming the possibility of extrapolation of the Brans-Dicke parameter data obtained from solar system experiments.

shows up an amplification of the hierarchy problem by a huge warp factor in one of the examples. In the second example this problem is partially circumvented by the Brans-Dicke parameter appearing in the power law warp factor.

One can study the linearized tensor fluctuations in order to localize gravity in the model. Keeping the classical approach to braneworld models, such a fluctuation is very important as well as the linearized tensor fluctuation is the analysis of the propagation of scalars [8, 15] in the background established. We will not do it here since the model is not complete in the sense that we made some specific choices for the constants. As we said before, a complete solution should contemplate the line element in the region $\rho < \rho_b$ (or $r < -r_b$), i. e., inside the brane. Normally, there are some standard approaches for the extension of solutions in the brane surface to the brane interior: it can be done, for example, by the analysis of the scalar propagator in the limit $\rho_b \rightarrow 0$ [8], or by some trick as the imposition of the \mathbb{Z}_2 symmetry at the brane surface [15]. Unfortunately, these two approaches are not possible in this model since it is based in a real topological defect, the global cosmic string, within the Brans-Dicke theory. In this vein, perhaps a more insightful approach is to find a solution splitting the spacetime into two regions (inside and outside the string) and finding the appropriated matching conditions, in a similar way of what it was done in the reference [24]. Besides, the moduli of the extra on-brane dimension can be stabilized by such junction conditions. This type of consideration should be taken seriously in the continuation of this research line.

To conclude, we should emphasize that the study of phenomenological implications of extra dimensions is a very interesting and promising field of research. In particular there is a great amount of work dealing with the possibility of some dependence of the standard model fields on extra dimensions, the curled and also the noncompact ones [25]. In the case of the dependence of the curled on brane dimension, for example, it is well known that the Kaluza-Klein tower, or specifically the lightest particle of such spectrum, can supply a good candidate to dark matter. Going into this direction, one can make an important and fundamental link between extra dimensional models and high energy phenomenology.

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